Algorithms and Data Structures
Spring 2018
Exercises for Unit 26

1. Let $G = (A, B; E)$ be a bipartite graph with $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_n\}$ of equal size n. The Edmonds matrix T for this graph is defined as an $n \times n$ matrix with $T_{ij} = 0$ if there is no edge between a_i and b_j and otherwise $T_{ij} = x_{ij}$ where x_{ij} is a variable.

   Prove that $\det T \neq 0$ iff G has a perfect matching (i.e. a matching of size n).

2. Three points a, b, c in the plane lie on a common line iff

$$\det \begin{pmatrix} 1 & a_1 & a_2 \\ 1 & b_1 & b_2 \\ 1 & c_1 & c_2 \end{pmatrix} = 0\,.$$

   How large an integer N would you choose so that selecting n points uniformly at random from the integer grid $\{1, \ldots, N\} \times \{1, \ldots, N\}$ produces with probability at least α a set S that has no 3 points on a common line?

   Extra question unrelated to today's material: If you are allowed to choose S from such a grid deterministically how "small" an N would you need?