Exercises for Unit 36

(3.) $\tau_e(x) = x \cdot \ell_e'(x)$

$$\ell_e^\tau(x) = \ell_e(x) + \tau_e(x) = \ell_e(x) + x \cdot \ell_e'(x)$$

$f^* :=$ optimal flow for latencies without taxes

use exercise 2: f equilibrium $\Leftrightarrow$ f minimizes Φ

$$\Phi(f) = \sum_{e \in E} \int_0^{f_e} \ell_e^\tau(x)\,dx$$

$$= \sum_{e \in E} \left(\int_0^{f_e} \ell_e(x)\,dx + \int_0^{f_e} x \cdot \ell_e'(x)\,dx \right)$$

$$= \sum_{e \in E} \left(\cancel{\int_0^{f_e} \ell_e(x)\,dx} + x \cdot \ell_e(x) \Big|_0^{f_e} - \cancel{\int_0^{f_e} \ell_e(x)\,dx} \right)$$

$$= \sum_{e \in E} f_e \cdot \ell_e(f_e) \rightsquigarrow \text{social cost}$$ for latencies without taxes, it is minimized by f^*

$\Rightarrow f^*$ is an equilibrium flow w.r.t. ℓ^τ

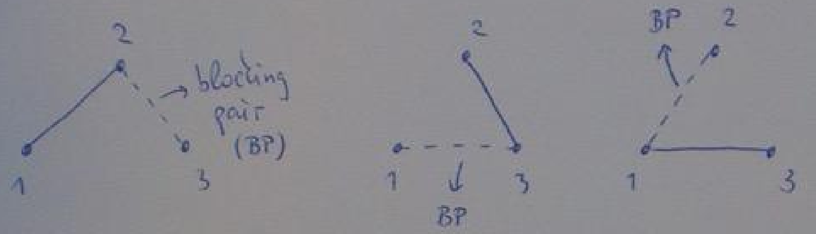
– this property also holds when we consider more general latency functions – as in exercise 2 (we just need ℓ_e to be convex ...)

Exercises for Units 37 & 38

6. a) $1 : 2 > 3$

$2 : 3 > 1$

$3 : 1 > 2$



b) Yes! Like for the bipartite version — pick the heaviest edge first, match the two vertices and remove them from the instance. We indeed do not have to consider them again because they cannot find a better partner. Repeat.